**In-plane anisotropy in spin-excitation spectra originating from chain states in YBa$_2$Cu$_3$O$_{6+y}$**

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We present a random-phase approximation (RPA)-based multilayer spin-susceptibility calculation for the trilayer YBa$_2$Cu$_3$O$_{6+y}$ (YBCO) system (including bilayer CuO$_2$ planes and uniaxial CuO chain layer) in the superconducting state. We show that the observed in-plane anisotropy in the spin-excitation spectrum of YBCO — which is often interpreted as an evidence for the electron nematic phase — can alternatively be explained via incorporating the uniaxial CuO chain's contribution. We demonstrate that the neutron spectra in YBCO is dominated by the contribution from the four-fold symmetric CuO$_2$ plane state as in other cuprates, however, it acquires an in-plane anisotropy via finite interlayer coupling with the chain state. The result rules out the claim that an electronic nematic phase is responsible for the pseudogap state in YBCO.

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I. INTRODUCTION

The formation of electronic liquid crystals is intimately related to two-dimensional anisotropy phenomena, which are of intense current interests in the field of strongly correlated electrons, semiconductors, 1D electron gases, 2 graphene 3 as well as in cold atom systems. 4 The surge of interests has recently been extended to various families of unconventional superconductors including cuprates, 5-9 pnictides, 10 Sr$_3$Ru$_2$O$_7$, 11 and heavy fermion URu$_2$Si$_2$, 12,13 The microscopic origin for the in-plane anisotropy in these superconducting (SC) systems which consistently accommodate tetragonal crystal symmetry is more interesting and exotic. A general question that underlies all these materials is whether the anisotropic structure is related to an emergent nematic electronic liquid crystal phase or merely comes from the subtle crystallographic distortion or something else.

In cuprates, pnictides, and heavy fermion URu$_2$Si$_2$ systems, this question has posed widespread research interests with the hope to explain their unusual normal state phases from which the SC state is arguably derived. In cuprates, the experimental evidences of the in-plane anisotropy have been taken seriously as a definite evidence for the presence of the electronic crystal liquid phase, 18,21 and also been extended to explain the so-called 1/8 doping problem where superconductivity is strongly quenched. 22 Here we provide a different mechanism for the origin of the in-plane anisotropy in YBa$_2$Cu$_3$O$_{6+y}$ (YBCO). We demonstrate that, even in the absence of any crystal anisotropy or 'nematic' ground state, an anisotropy is introduced to the SC CuO$_2$ plane state via the interlayer electronic coupling with the quasi-1D CuO chain bands.

*Experimental implications of anisotropy in cuprates:*  
The CuO$_2$ plane state is present in all cuprates, and is arguably responsible for most of the exotic low-energy phases which vary dramatically within the same family. A stripe-like modulation has been well established in La$_{2-x}$Sr$_x$Ba$_2$CuO$_4$ (LS/BCO), 23 Bi$_2$Sr$_2$CaCu$_2$O$_{8+y}$ (BSCCO) 24, Cu$_{2−x}$Na$_x$CuO$_2$Cl$_2$, 25 and HgBa$_2$CuO$_{4+δ}$ (HBCO). 26 On the other hand, most evidences for the anisotropic structure have been obtained in YBCO via transport, 5 Nernst, 9 and inelastic neutron scattering (INS) measurements 6, with some recent indication of this phase in BSCCO from scanning tunneling microscopy (STM). 7,8 This material variant pseudogap properties raise an essential question: *Is the mechanism of pseudogap different in different cuprates, or such differences merely arise from the material dependent electronic structure?* By closely looking at the YBCO crystal, we indeed notice that this system hosts both
quasi-2D CuO$_2$ layers and quasi-1D CuO chain layers, as shown in Fig. 1(a). The FSs measured by angle-resolved photoemission spectroscopy (ARPES) in the underdoped [Fig. 1(b)] and overdoped [Fig. 1(c)] region exhibit that while a pseudogap is opened in the CuO$_2$ bi-layer bands, but the 1D CuO chain FS remains ungapped at all finite dopings.\textsuperscript{28,31} Additionally, a visual estimation to the Fermi arc seen in the underdoped case does not reveal any significant presence of four-fold symmetry breaking, thus indicating that the origin of pseudogap is not directly related to the observed anisotropy in this system. On the other hand, YBCO being a polar compound, there exists a charge transfer between the layers as a result of the electric fields generated by the polar unit cell.\textsuperscript{27} We compute the INS spectra by realistic multi-layer BCS-RPA calculation to demonstrate that the interlayer coupling between the chain and plane layers stimulates a two-fold symmetry in the electronic states even when the isolated bulk planer states do not intrinsically involve any spontaneous four-fold symmetry breaking.

The rest of the paper is organized as follows. In Sec. II, we introduce the tight-binding model for the trilayer YBCO system and the corresponding multi-layer spin-excitation calculation. The results for the static spin-susceptibility at zero energy and the role of two plane FS are and the chain FS are given in Sec. III. In the SC state, the origin of the in-plane anisotropy in the dynamical spin-excitation spectrum is demonstrated in Sec. IV. The doping and energy dependence of the FS nesting and the in-plane anisotropy is calculated and discussed in the context of YBCO phase diagram in Sec. V. Finally we discuss and conclude in Sec. VI.

II. TIGHT-BINDING MODEL

To realistically model the low-lying electronic states of YBCO, one requires a minimum of three band model to incorporate the bi-layer splitting and the chain band, coming from CuO$_2$-CuO$_2$-CuO trilayers.\textsuperscript{30} In each isolated layer, the single electron hopping between Cu $d_{x^2-y^2}$ orbitals is mediated by O atoms, which is captured by effective tight-binding hopping parameters. For the plane and chain layers, it is sufficient to consider up to third and next nearest neighbor hoppings, respectively [see Fig. 1(a)], which construct the isolated layer dispersions $\xi_{(p/c)k}$ as\textsuperscript{12}

\[
\begin{align*}
\xi_{pk} &= -2t_{c}(c_x + c_y) - 4t'_{c}c_x c_y - 2t''_{c}(c_{2x} + c_{2y}) - \mu_{p}, \\
\xi_{ck} &= -2t_{c}c_y - \mu_{c}.
\end{align*}
\]

(1)

(2)

Here $c_{x/y} = \cos(\alpha k_{x/y})$ and $\mu_{p/c}$ are the chemical potentials for the plane and chain bands, which encode the onsite potential imbalance and other crystal field effects between the two layers. We consider the interlayer coupling within single electron tunneling matrix-element $t_{pp}$ and $t_{cp}$, which obey periodic boundary condition along the c-axis. Therefore the final Hamiltonian in the trilayer basis is,

\[
H_k = \begin{pmatrix} \xi_{pk} & t_{pp} & t_{cp} \\ t_{pp} & \xi_{pk} & t_{cp} \\ t_{cp} & t_{cp} & \xi_{ck} \end{pmatrix}.
\]

(3)

Note that while isolated plane dispersion $\xi_{pk}$ obeys four-fold symmetry, the total Hamiltonian in Eq. 3 breaks this symmetry due to its coupling with the chain band via $t_{cp}$. In other words, all eigenvalues and eigenstates including that for the plane states are two-fold symmetric. The computed spectral weight function on the 2D momentum space at the Fermi level is plotted in Fig. 1(d) at a representative doping $x = 0.15$ [this choice of doping represents YBCO$_{0.45}$ at which the neutron scattering data is compared below], which agrees well with the corresponding FS trend obtained in ARPES data [Figs. 1(b)-(c)].\textsuperscript{31}

A. Multilayer BCS susceptibility

The superconductivity is taken to be $d$-wave like for all three bands, where the value of the SC gap amplitude is constrained by the experimental value\textsuperscript{31,33} $\Delta = 34$ meV.\textsuperscript{34} In the SC state, the layer-dependent BCS spin-susceptibility tensor is computed from

\[
\begin{align*}
\chi^{j,j'0}_{\alpha\nu}(q, i\omega_m) &= -\frac{1}{4\pi N} \sum_{k,n,\nu',\nu''} M^{\nu'\nu''}_{i\nu,j}\langle k, q | G_{\nu'}^{\nu''}(k + q, i\Omega_n + i\omega_m) \rangle, \\
\text{Tr} \left[ G_{\nu'}^{\nu''}(k, i\Omega_n) G_{\nu''}^{\nu'}(k + q, i\Omega_n + i\omega_m) \right].
\end{align*}
\]

(4)

For each band, $G$ is the $2 \times 2$ Green’s function, made of normal and anomalous parts in the Nambu-Gorkov’s notation.\textsuperscript{35} The interlayer coupling matrix element is $M^{\nu'\nu''}_{i\nu,j\nu'}(k, q) = \phi^{\nu}_{j'}(k)\phi^{\nu'}_{i\nu}(k + q)\phi^{\nu'}_{i\nu^'}(k + q)$, where $\phi^{\nu}_{j'}(k)$ is the eigenstate of band $\nu$ or layer $i$. In this notation, $\nu (\nu')$ is the initial (final) eigenstate, consists of hybridization between $i$ and $j$ layers, and $\phi$ is the corresponding eigenvector.

RPA:– Finally, we incorporate the many-body interaction within random-phase approximation (RPA) as $\chi(q, \Omega) = \chi(q, \Omega) [1 - U(q, \Omega)]^{-1}$ (the symbol tilde represents tensor). Since, we consider only the $d_{x^2-y^2}$-orbital for all three layers, no inter-orbital couplings such as Hund’s coupling are involved here. We include intra-layer interaction for plane and chain as $U_p$ and $U_c$, respectively and the inter-layer one between plane-plane and chain-plane layers as $U_{pp}$ and $U_{cp}$.\textsuperscript{43}

III. ANISOTROPY IN STATIC SUSCEPTIBILITY AND THE ROLE OF FS NESTING

Figure 2 shows the relevant components of the static RPA susceptibilities in the normal state and their corresponding FS nestings. The intraplanar component, in Fig. 2(a), obtains a leading incommensurate nesting at $q^{np}_{0} = (\pi(1 - \delta_a), \pi)$ and $q^{np}_{0} = (\pi, \pi(1 - \delta_b))$, where $\delta_a$ and $\delta_b$ measure the degree of incommensurability along $a$ and $b$ bond directions. Comparing these two nesting peaks, we immediately find that there is a prominent anisotropy present in both intensity and magnitude of $\delta_a$ and $\delta_b$. The origin of this anisotropy can be traced back to the FS anisotropy of the plane band shown by blue lines in Fig. 2(e). This anisotropy in the plane state is introduced via its hybridization with the chain layer (recall that the
isolated plane dispersion, $\xi_{\mathbf{q} \mathbf{k}}$, itself obeys four-fold symmetry. The bi-layer splitting in the plane bands is clearly visible near $\mathbf{k} = (0, \pm \pi)$ points while near $\mathbf{k} = (\pm \pi, 0)$, it mixes with the chain bands. The nesting between these split bands, $\mathbf{q}_{pp'}$, produces a susceptibility peak in the vicinity of $\mathbf{q} \sim \Gamma$, as seen in Fig. 2(b). The intra-chain and chain-plane components of the susceptibility in Figs. 2(c) and 2(d), respectively are naturally dominated by the quasi-1D components. Summing over its components at each $\mathbf{q}$ and $\omega$, we see that the full susceptibility accommodates the coexistence of several nestings with relative intensities, see Fig. 2(f). The competition between various instabilities originating from the leading nesting is strongly doping dependent, and hence on the FS topological changes as discussed below in Fig. 4.

IV. ANISOTROPY IN SPIN RESONANCE AND SUPERCONDUCTIVITY

INS directly measures the imaginary part of the spin susceptibility as a function of $\mathbf{q}$ and $\omega$. Within the standard itinerant picture of unconventional superconductivity, a spin resonance develops in the SC state due to the sign-reversal of the SC order parameters at the FS nesting vectors. On the basis of this theory, only $\mathbf{q}_{pp}$ vector contributes to the BCS susceptibility, while the other nesting vectors disappear or become suppressed in the SC state (of course, the inelastic scattering activates other possible scattering channels, however, their intensities are shown earlier to be comparatively less). The computed spin resonance spectra plotted along $\alpha$- and $\beta$-bond directions in Figs. 2(a) and 2(b), respectively, reveal that the anisotropy between $\delta_{\alpha}$, $\delta_{\beta}$ acquires energy dependence due to the $d$-wave nature of the SC gap. At zero energy, both $q_{pp}$ touches the nodal points, where $\delta_{\alpha/\beta}$ attain their maximum amplitudes. With increasing energy, $q_{pp}$ approach towards $Q$, and thereby reducing the values of $\delta_{\alpha/\beta}$. Finally, at $\omega = 40$ eV, both $q_{pp}$ march at $Q = (\pi, \pi)$, the intensity of $\chi''$ becomes maximum and a resonance peak appears. Although the anisotropy disappears at the resonance in its magnitude ($\delta_{\alpha} = \delta_{\beta} = 0$), but it remains present in the intensity as can be seen from Fig. 2(e), due to overlap matrix-element effect between plane and chain layers, see Eq. 4. In fact, the anisotropy becomes reversed at and above the resonance energy, as demonstrated in Fig. 3(c). The results agree well with the experimental data of YBCO$_{4.5}$, presented in Figs. 2(f)-

FIG. 2. (Color online) (a)-(d) Various representative components of the static RPA susceptibility in the normal state at $x=0.15$. The dominant nesting in each component is depicted by arrows. The arrows of same color are overlayed on the corresponding FS in (e). (c) The FS obtained from Fig. 1(d). (f) Total RPA susceptibility (summing over all components).

FIG. 3. (Color online) (a)-(b) Computed imaginary part of the total susceptibility, $\chi''$, in the SC state plotted along $(0, \pi) \rightarrow (2 \pi, \pi)$ in (a), and $(\pi, 0) \rightarrow (\pi, 2 \pi)$ in (b) ($\alpha$- and $\beta$-bond directions, respectively). (c) Several representative constant energy cuts of $\chi''$ are shown as a function of $\mathbf{q}$ along $\alpha$- and $\beta$-bond directions. Each spectra are shifted vertically by a constant value for visualization. The peak position of each spectrum is marked by a small vertical line. (d)-(e) Two representative constant energy profiles of $\chi''$ shown below and near the resonance, respectively. (f) Two representative momenta cuts through (e) (arrows) shown near $Q$, and compared with corresponding experimental data (symbols of same color). The intensities of the theoretical and experimental data are normalized arbitrarily to ease the comparison. The dashed line through the experimental data is a guide to the eye. (g)-(h) The constant energy experimental profiles.
It is noteworthy that the $45^\circ$ of the INS spectra as a function of doping is observed earlier in La-based materials where such rotation occurs at the beginning of the SC dome around $x=0.05,38$ while in the present case of YBCO, this crossover doping coincides with the optimal doping of this compound. (3) The doping gradually increases intensity at other small nesting vectors $q_{pp}$ near $\Gamma$ point, which all participate in nesting the van-hove singularity (VHS) region of the quasi-2D plane bands, and the quasi-1D FSs of the chain bands, respectively, as defined in Fig. 2(e). With hole doping, these flat FSs move towards each other [compare FSs at $x=0.025$ and $x=0.30$ in the insets to Fig. 4(e)], and thereby enhance nestings. Similar VHS nesting has been theoretically predicted to be present in the single-layer BSCCO band structure,39 and observed in the quasiparticle interference (QPI) pattern of the same compound as probed by STM40 (in YBCO, this nesting additionally breaks four-fold symmetry). However, as mentioned earlier, these nestings do not survive in the SC state, because they do not involve sign-reversal of the $d$-wave SC gap, and thus cannot be detected by INS. (4) Above the optimal doping, the maximum intensity at $\delta_a$ jumps suddenly from the $(\pi, \pi)$ region (plane bands) to the chain band region at $q_{pp}$ [dictated by arrows in Figs. 4(d) and 2(e)] as plotted by square symbols in Fig. 4(e). As the chain band is 1D, $q_{pp}$ only appears along 1D, while the incommensurate peak along the $a$-bond direction continues to be dominated by plane states. This is why, the anisotropy between $\delta_a$ and $\delta_b$ becomes reversed over the optimal doping.

V. DOPING DEPENDENCE OF ANISOTROPY

Finally, we study the doping evolution of the anisotropy in YBCO, and compare it with the YBCO phase diagram in Fig. 4. In the extreme underdoped region $x=0.025$, the leading static instability in $\chi(q, \omega=0)$ commences at $q_{pp}$ with both the magnitude and intensity of $\delta_a$ being larger than those of $\delta_b$, as seen in Fig. 4(a). With increasing doping, we notice several interesting characteristic evolution of the susceptibility: (1) Both $\delta_a$ and $\delta_b$ increase almost monotonically with doping, however the degree of anisotropy or the difference $\delta_a - \delta_b$, does not have significant doping dependence up to $x \sim 0.15$, see Fig. 4(e). (2) With further hole doping, the incommensurate peaks along the bond directions $q_{pp}$ and $q_{pp}^{\parallel}$ begin to split. Above $x=0.15$, the splitting becomes sufficiently large so that the maximum intensity in the vicinity of $Q$ does not remain aligned along the bond directions, but rotates by $45^\circ$ to the diagonal directions [see Figs. 4(c)-(d)].

VI. DISCUSSION AND CONCLUSION

Does anisotropy cause nematic pseudogap order? Coincidentally, the crossover of the leading instability from the plane to the chain band occurs at the doping where evidences are mounting in YBCO that the pseudogap also disappears.26,28,33 On the basis of these results as well as numerous evidences for the presence of FS reconstruction in the pseudogap region of YBCO, we conjecture that if pseudogap arises from a FS nesting, the planar nesting $q_{pp} \sim (\pi, \pi)$ that dominates in this doping region, is most likely be the one.20,41 The leading nesting at $q_{pp}$ also prohibits the possibility of stabilizing a charge density wave (CDW) in the 1D chain bands via Peierls instability at $q_{pp}$ in the pseudogap state. CDW can at most be present as a secondary order.44 Recalling the ARPES results presented in Fig. 1(b)-(c), our results are consistent with the observed ungapped chain bands at all finite dopings.

Conclusions:- In summary, we provide a microscopic model for the trilayer YBCO superconductor to explain the observed in-plane anisotropy in the spin-excitation spectrum of this compound. We calculate multi-layer BCS susceptibility within RPA formalism to show that while the isolated planar electronic states intrinsically obey four-fold rotational symmetry, but its observables exhibit a reduced two-fold symmetry due to inter-layer coupling with the quasi-1D chain bands. The computed INS spectra exhibit considerable anisotropy in its incommensurate structure which varies with energy up to the resonance energy scale, in quantitative agree-
ment with experimental data at the same doping. We also present verifiable prediction that the anisotropy becomes reversed above optimal doping in YBCO. We conclude that an incommensurate order with planar nesting around $(\pi, \pi)$ is the leading nesting as in other cuprates, while the anisotropy is a secondary effect coming form the chain state. Our results rule out the earlier postulate that nematic order may be the origin of pseudogap phase in YBCO.

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[14] In YBCO, it is observed that a nematic phase forms around $T^* ∼150$ K (YBCO), which is higher than the ‘pseudogap’ temperature $T^* ∼200$ K for this compound, and also higher than the onset of orthorhombic distortion or the low-temperature orthorhombic phase in LSCO and in pnictide [J.-H. Chu, Science 329, 824 (2010)].
[27] It is known that YBCO crystals mostly cleave between the CuO chain and the BaO layers, exposing the CuO chain to the surface.
[32] The parameters are obtained after fitting to the experimental FS as $\nu(\mathbf{k}, \omega_n) = \nu(\mathbf{k}, \omega_n) = \nu(\mathbf{k}, \omega_n) = \nu(\mathbf{k}, \omega_n) = \nu(\mathbf{k}, \omega_n)$.
[34] The widely used proximity model for YBCO suggests that the d-wave superconductivity is induced to the chain layer from the plane one due to interlayer coupling [D. K. Morr and A. V. Balatsky, Phys. Rev. Lett. 87, 247002 (2001)]. We take the gap amplitude to be same for all bands to reduce the number of parameters.
[35] The components of the BCS Green’s function are $G_{11,22}(\mathbf{k}, \omega_n) = (\omega_n + \xi_{\mathbf{k}})/\omega_n^2 + E_{\mathbf{k}}^2$ and $G_{12}(\mathbf{k}, \omega_n) = G_{21}(\mathbf{k}, \omega_n) = \Delta_{\mathbf{k}} = \pm \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$, where the SC quasiparticles are $E_{\mathbf{k}} = \pm \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$. (G12 is sometimes denoted by F.) We perform Matsubara frequency
summation over the fermionic index $n$, while the bosonic Matsubara frequency is treated by taking analytical continuation to the real axis as $i\omega_m = \omega + i\delta$ ($\delta$ is small broadening).


[43] Denoting the intra-layer interactions for plane and chain as $U_p$ and $U_c$, respectively and the inter-layer one between plane-plane and chain-plane layers as $U_{pp}$ and $U_{cp}$, the nonvanishing components of the $\bar{U}$ tensor are $\bar{U}_{ii}^{ii} = U_i$, $\bar{U}_{ij}^{ij} = U_{ij}$ (where $i, j = p, c$). The values of each components of $\bar{U}$ are constrained by their corresponding components of $\bar{\chi}$ to avoid an instability, see Fig. 2, and the obtained values are $(U_p, U_c, U_{pp}, U_{cp}) = (3.2, 0.5, 0.4)$ in eV.