# Term paper for PH364: Topological Phases of Matter

1) **Solution of Landau level problem in a tight-binding model. This is also known as Hofstadter butterfly model or TKNN theory in tight-binding model with slight modification.**

   Imagine a 2D square lattice in the xy-plane, and a magnetic field penetrating perpendicular to it (i.e. **B** = $B_z$). The electrons are not interacting. *Numerically solve the Landau level problem with tight-binding method.*
   *(a) Plot bulk bands as a function of k.*
   *(b) Plot edge states.*
   *(c) Plot the wavefunction for the edge states*

   Refs: i) D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs
   Phys. Rev. Lett. 49, 405 1982
   ii) Bernevig's book, Chapter 5.
   ii) http://www.physics.iisc.ernet.in/~qcmjc/talk_slides/QCMJC.2013.08.22_Yinghai.pdf

2) **Solve edge state problem in 2D/3D topological insulators.**
   (a) Solve the continuum model for 2D and 3D for a time-reversal breaking Chern insulator and a time-reversal invariant Z2 topological insulator
   (b) Repeat the calculation in a finite lattice tight-binding model with open boundary condition in any one direction. Plot the edge and quantum well states and demonstrate bulk-boundary correspondence.
   (c) Discuss the topological invariants for the two models.

   References:
   i) Bernevig's book.
   ii) B. Andrei Bernevig, Taylor L. Hughes, Shou-Cheng Zhang, Science 314, 1757 (2006)
   iii) X.-L. Qi, S. C. Zhang, RMP 83, 1051 (2011).

3) **Topological crystalline insulator (TCI)**
   a) Discuss what is a TCI.
   b) Reproduce the derivation of Z2 invariant for TCI
   c) Either solve for the Z2 invariant or Mirror Chern number for a model TCI Hamiltonian
   References:
   i) Liang Fu, Phys. Rev. Lett. **106**, 106802 – Published 8 March 2011
   ii) Timothy H. Hsieh, Nature Communications volume3, Article number: 982 (2012)
   iii) Yoichi Ando, Liang Fu, arXiv:1501.00531.

4) **Axion insulators**
   a) What is an axion insulator?
   b) Derive Eq. 3 of Ref. i below, or Eq. 3 of Ref. ii. (Same equation).
   c) Plot axion angle as a function of magnetic moment for a different Hamiltonian that I will give you.

References
- i) Rundong Li, et al. Nature Physics 6, 284 (2010).
- ii) Akihiko Sekine, and Kentaro Nomura, arXiv:1401.4523.
- iii) Jing Wang, et at PRL 106, 126403 (2011).

5) **Topological superconductor**

Kitaev model is a topological superconductor in 1D.

a) Give examples of 2D topological superconductors for s-wave and p-wave pairings
b) How to obtain the topological invariant for such a system
c) What is the edge mode for a topological superconductor?

References:

i) Bernevig's book.
ii) X.-L. Qi, S. C. Zhang, RMP 83, 1051 (2011)
iii) Masatoshi Sato, Yoichi Ando, Rep. Prog. Phys. 80, 076501 (2017); arXiv:1608.03395
iv) Xiao-Liang Qi, et al, PRL 102, 187001 (2009)
v) Fan Zhang, C. L. Kane, and E. J. Mele, PRL 111, 056402 (2013); PRL 111, 056403 (2013)

**6) Floquet topological insulator**

a) What is a Floquet Hamiltonian?
b) How non-trivial Berry phase arises in a Floquet Hamiltonian
c) Demonstrate the result with at least one example case.

References: (i) N H. Lindner, Gil Refael & V Galitski, Nature Physics volume 7, pages 490–495 (2011)
(ii) Jérôme Cayssol, Balázs Dóra, Ferenc Simon, Roderich Moessner, arXiv:1211.5623
(iii) Takuya Kitagawa, Erez Berg, Mark Rudner, and Eugene Demler, PRB 82, 235114 (2010)

7) **3D Dirac and Weyl semimetals**

a) What is a 3D Dirac Hamiltonian and how does it split into Wey cones?
b) Demonstrate the topological properties for these cases with at least one example
c) Fermi arc on the surface of Weyl fermions.

References: (i) N.P. Armitage, E. J. Mele, Ashvin Vishwanath, Rev. Mod. Phys. 90, 15001 (2018)
(ii) Oskar Vafek, Ashvin Vishwanath, arXiv:1306.2272
(iii) Gábor B. Halász, Leon Balents, Phys. Rev. B 85, 035103 (2012)
(iv) A.A. Burkov, arXiv:1704.06660

**8) Dirac nodal line semimetal**

a) Discuss the definition and criterion of Dirac nodal line semimetals in 2D and 3D
b) Discuss the usual symmetry properties as mentioned in the class and also the crystalline and translational symmetry in this ocontext
3) Plot the bulk band structure and nodal lines, their topological properties and the edge/surface states

References:
- i) Chen Fang, Hongming Weng, Xi Dai, Zhong Fang, Chinese Phys. B 25, 117106 (2016); arXiv:1609.05414
- ii) A. A. Burkov, M. D. Hook, and Leon Balents, Phys. Rev. B 84, 235126 – Published 20 December 2011

**9) Higher order topological insulators and corner states**

a) Define higher order topological invariants and corner states in 2D and 3D

b) Symmetry properties and topological protections

c) Plot the bulk and corner states in at least one example case

References:


i) Frank Schindler, Ashley M. Cook, Maia G. Vergniory, Zhijun Wang, Stuart S. P. Parkin, B. Andrei Bernevig, Titus Neupert, Science Advances 01 Jun 2018: Vol. 4, no. 6, eaat0346

ii) Eslam Khalaf, arXiv:1801.10050v4

iii) Frank Schindler, et al, Nature Physics volume 14, pages918–924(2018) See supplementary information


10) **Topological classification table**

Due to the combination of time-reversal, charge conjugation, and chiral symmetries, there are ten difference classes of topological phases possible in all dimensions. Take 2D

a) Discuss the symmetry properties of these three symmetries.

b) Discuss the theory behind the topological classifications.

c) Give one example (model) for all ten classes of topological phases

References


i) Ching-Kai Chiu, et al, Reviews of Modern Physics, volume 88, july–september 2016

ii) Shinsei Ryu1,6, Andreas P Schnyder2,3,6, Akira Furusaki4 and Andreas W W Ludwig5, New Journal of Physics 12 (2010) 065010 (60pp)

iii) Jeffrey C. Y. Teo and C. L. Kane, Physical Review B **82**, 115120 2010.

iv) Andreas P. Schnyder,[1] Shinsei Ryu,[1] Akira Furusaki,[2] and Andreas W. W. Ludwig[3], Physical Review B **78**, 195125 2008


**11) Triple point semimetals**

a) Define triple point semimetals (tree fold degeneracy) and the topological phases

b) Symmetry properties

c) Plots bulk and surface bands with one example

References:


i) Barry Bradlyn, et al, Science 05 Aug 2016: Vol. 353, Issue 6299, aaf5037

ii) Ziming Zhu, Georg W. Winkler, QuanSheng Wu, Ju Li, and Alexey A. Soluyanov, Phys. Rev. X 6, 031003 – Published 7 July 2016

iii) Hongming Weng, Chen Fang, Zhong Fang, and Xi Dai, Phys. Rev. B 93, 241202(R) – Published 23 June 2016


12) **Chiral anomaly**

a) What is a chiral anomaly?

b) Derive the formula for chiral anomaly in 1D and 3D Weyl fermion systems

c) Derive the formula for resistivity versus magnetic field for a chiral anomaly

References:


i) Ming-Che Chang, Lecture notes: Chapter 9, 14.

ii) Pallab Goswami[1] and Sumanta Tewari, arXiv:1210.6352v3

iii) Pavan Hosur, X. L Qi, arXiv:1309.4464v1

iv) A.A. Burkov, Annu. Rev. Condens. Matter Phys. 2018.9:359-378, J. Phys.: Condens. Matter **27** (2015) 113201 (18pp)

**13) Experimental signatures of Berry phase**

a) Derive the formula for how berry phase modifies the Shubnikov-de Haas van Alphen oscillations and Landau level. In other words, how to obtain the value of Berry phase from these measurements.
b) Show that a system with finite Berry phase (such as graphene or Dirac/Weyl semimetals) give negative magnetoresistance.
c) Discuss the mechanism of anti-localization.

References:

(i) Di Xiao, Ming-Che Chang, Qian Niu, Rev.Mod.Phys.82:1959-2007(2010).
(ii) D. T. Son and B. Z. Spivak, Phys. Rev. B 88, 104412 (2013)
(iii) Dong-Xia Qu, Y. S. Hor, Jun Xiong, R. J. Cava and N. P. Ong, Science **329**, 821 (2010),

14) **Topological Kondo Insulator**

a) What is a Kondo and mixed valence insulator?
b) How topological phase transition occurs in a Kondo insulator?
c) Properties of edge state.

References: (a) Maxim Dzero, Jing Xia, Victor Galitski, Piers Coleman, arXiv:1506.05635
(b) Maxim Dzero, Kai Sun, Victor Galitski, Piers Coleman, PRL 104, 106408 (2010)
(c) Maxim Dzero, Kai Sun, Piers Coleman, Victor Galitski, PRB 85, 045130 (2012)
(d) Victor Alexandrov, Maxim Dzero, Piers Coleman, PRL 111, 226403, (2013)