PH205:Mathematical Methods of Physics   Total Marks:100

Date:25-10-2016   Quiz-2   Maximum Time: 1.5hours

---

1. Tensor: Tensor is a generalization of the ideas of scalar, vector and matrix, to quantities with many more indices (rank). The location of the indices in the tensor has special meaning: A^i is called a contravariant tensor of rank 1 and A_i is called a covariant tensor of rank 1. The laws of nature are covariant, i.e. a valid tensor, the tensor equation, and/or the symmetry of the tensor is something that remains preserved under coordinate transformation. The most trivial tensors we use are Kroenecker delta δ_i^j (a tensor of rank (1,1)) or Levi-Civita ϵ_{ijk} (a tensor of rank 3). They follow an important identity

$$\epsilon_{ijk}\epsilon^{lmk} = \delta_i^l \delta_j^m - \delta_i^m \delta_j^l$$

1a If S, is a covariant symmetric tensor of rank-2, and A is a contravariant, antysymmetric tensor of rank 2, prove that their corresponding symmetries are preserved. (5+5)

1b For the same S, and A tensors, prove that their contraction gives zero. (5)

1c Prove that $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - \mathbf{B}(\nabla \cdot \mathbf{A}) - (\mathbf{A} \cdot \nabla)\mathbf{B}$, for A, B are tensors of rank-1. (10)

2. If a function $f(z)$ fails to be analytic at a point, such a point is called a singular point. A point 'a' is called an isolated singularity if the function is analytic in the neighbourhood of the point ($f(z)$ is analytic $\forall z$ such that $|z - a| < \delta$ where δ is infinitesimally small). Otherwise, it is called a non-isolated singularity. If the principal value of $f(z)$ at in $\lim_{z\to a} f(z)$ is finite while a is a singular point then a is said to be a removable singularity. Finally, if a function $f(z)$has an infinite number of singularities at $z = a$ i.e., there are infinite negative degree terms in the Laurent series then, a is called an essential singularity.

Cauchy-Riemann equation is one of the building blocks of complex analysis. This says that the derivative of a complex variable $f(z)$ with respect to x or y should be the same (where $z = x + iy$).

2a Consider the function $f(z) = (z - a)^p$, where a is a positive constant. Evaluate if $f(z)$ is analytic everywhere, or have any type of pole or Branch-point etc. for the following cases (show your explicit calculations and arguments as required): (i) $p > 0$ and integer. (ii) $p > 0$ and fraction. (iii) $p < 0$ and integer. (iv) $p < 0$ and fraction. (5+5+5+5)

2b (i) Find the series expansion (Taylor, Laurent, as applicable) of $\frac{e^z}{z^3}$. (ii) Identify the pole (s). (iii) Evaluate $\oint_C \frac{e^z}{z^3} dz$ in a unit circle. (15+5+10)

3.- (Answer any one) (25)
Use residue theorem to evaluate: **(a)** $\int_0^\infty \frac{x^2+1}{x^4+1}$, or **(b)** $\int_{-\pi}^{\pi} \frac{1}{1+\sin^2\theta} d\theta$.